CyberChallenge.IT 2025 - Pretest
Commented solutions

Contents

1 Question 1

1.1 Question

An `8x8` board is filled with the letters A, B, C, D. The board is called *cool* if every row, every column and the two main diagonals (the one starting top left and ending bottom right, and the one starting top right and ending bottom left) is palindrome (reads the same in both directions). How many `8x8` *cool* boards are there?

*Note: boards obtained from rotations or reflections of other boards are considered **distinct**.*

1.2 Answers

(A) 2^{16}

(B) 2^{32}

(C) 2^{48}

(D) 2^{64}

1.3 Proposed solution

The correct answer is `(B)` 2^{32}.

Let's point out that each *cool* board is symmetrical with respect to its vertical and horizontal axis. This means we can just count how many ways we can fill the `4x4` top left cells of the board, and then the `8x8` board obtained by flipping it will be *cool*. We don't have any other restriction on the `4x4` grid, so we can fill it in 4^{16} ways: 16 cells that can be independently filled with 4 values.

The answer is therefore $4^{16} = 2^{32}$.

2 Question 2

2.1 Question

A curious student talks with her CyberChallenge.IT tutor. “I heard you have three daughters. How old are they?”. The tutor answers: “Well, the product of my daughters’ ages is 72, and the sum of their ages is the number of the room we are in”. The student writes a few things down and then says: “I don’t have enough information”. The tutor exclaims: “Ah, of course! I forgot to tell you that my oldest daughter is my favourite!”. The student now knows all their ages. How old is the oldest daughter?

2.2 Answers

(A) `6`

(B) `8`

(C) `9`

(D) `12`

2.3 Proposed solution

The correct answer is `(B) 8`.

The product of the daughters’ ages is 72. According to the conversation, the knowledge of both the product and the sum of their ages does not uniquely identify them. Therefore, there must be two factorizations of 72 in 3 positive numbers with the same sum. We can start by making a list of all possible factorizations of 72 in 3 positive numbers, together with their respective sums. Since the question asks for the oldest daughter’s age, we also know that the maximum number in a factorization triplet must be one of the answers, so we can reduce them to the list below:

1. $(2, 6, 6) \to 14$
2. $(3, 4, 6) \to 13$
3. $(3, 3, 8) \to 14$
4. $(1, 8, 9) \to 18$
5. $(2, 4, 9) \to 15$
6. $(1, 6, 12) \to 19$
7. $(2, 3, 12) \to 17$

Only factorizations 1 and 3 have the same sum, so one of them must be the correct one. We must exclude the first one, because the number 6 is present twice, but there is only one oldest daughter; the correct answer is then 8.

3 Question 3

3.1 Question

Riccardo extracts three integer numbers. The numbers are drawn between 1 and 2025 (both included), uniformly at random, and they can be repeated. Riccardo does not extract the number 1729. What is the probability that the product of the three extracted number is odd?

3.2 Answers

(A) `1/8`

(B) `7/8`

(C) `1/2`

(D) `(1012/2025)`^{3}

3.3 Proposed solution

The correct answer is `(A) 1/8`.

In order for the product to be odd, each multiplied number must be odd. For each draw, Riccardo has $\frac{1}{2}$ probability to extract an odd number: there are 1013 odd and 1012 even numbers between 1 and 2025, but we know one of the odd ones is never extracted (1729). Since the draws are independent from each other, the probability each draw is odd is therefore $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$.

4 Question 4

4.1 Question

You are facing a challenge vending machine. The vending machine has three buttons that sell respectively a Misc challenge, a Network challenge or a random one between Misc and Network. The wiring is made wrongly, so that no button does what it claims to do. If each challenge costs 1€, how many euros do you need to find out which button does what?

4.2 Answers

(A) `1`

(B) `2`

(C) `3`

(D) `It cannot be determined`

4.3 Proposed solution

The correct answer is `(A) 1`.

We can start by pressing the button that claims to sell a random challenge. From the problem description, we know no button does what it claims, so we know this button actually sells only Misc or only Network challenges. Without loss of generality, let's suppose we are given a Misc challenge (the reasoning is the same if we get a Network one). Then we know this button is the one that sells Misc challenges only. No button does what it claims, so the button claiming Network only challenges cannot sell Network only challenges, therefore, it must sell random challenges. We are then left with the button claiming Misc only challenges, which sells Network only challenges.

With this strategy, we can link each button to its product with just 1€; it is obviously not possible with 0€, as there are multiple ways to make the wiring with the given constraint.

5 Question 5

5.1 Question

Alice does not remember what day is today. However, she knows that:

- Today is neither Tuesday nor Friday
- Tomorrow is neither Tuesday nor Friday
- Yesterday was not Sunday
- Wednesday was not the day before yesterday, neither was Tuesday

Knowing that exactly one of the statements in the list is false, which day of the week is today?

5.2 Answers

(A) `Monday`

(B) `Tuesday`

(C) `Thursday`

(D) `Friday`

5.3 Proposed solution

The correct answer is `(B) Tuesday`.

Let the four sentences be Sentence 1, . . . , Sentence 4, in order. We can build a truth table to check how many sentences are false for each day of the week.

	1	2	3	4
Monday	True	False	False	True
Tuesday	False	True	True	True
Wednesday	True	True	True	True
Thursday	True	False	True	False
Friday	False	True	True	False
Saturday	True	True	True	True
Sunday	True	True	True	True

The only day of the week with exactly one false statements is Tuesday.

6 Question 6

6.1 Question

Consider the following function.

```
function encrypt(m, len):
  res = [0]*len # this variable is set to [0, 0, ..., 0] where the array has length len
  for i = 0,1,...,len-1:
    res[i] = chr((ord(m[i]) * ord(m[(i+1)%len])) % 26)
  return res
```

Where `ord` is the function that maps A to 0, B to 1 and so on, while `chr` is its inverse function. How many strings `m` (even meaningless) are there such that `encrypt(m, 10) = "BBBBBBBBBB"`?

6.2 Answers

(A) `1`

(B) `12`

(C) `13`

(D) `26`

6.3 Proposed solution

Note: during the test option (B) had a typo, leading to no correct answers for this question. Due to this issue, the question was considered correct for all participants.

The correct answer is `(B) 12`.

Let $c_0, \dots, c_9$ be the values of the characters of m. Since `ord('B')` $= 1$, we can build the following equations:

$$\begin{aligned} c_0 \cdot c_1 &= 1 \pmod{26} \\ c_1 \cdot c_2 &= 1 \pmod{26} \\ &\vdots \\ c_9 \cdot c_0 &= 1 \pmod{26} \end{aligned}$$

1 has no common factor with 26, so each character c_i also has no common factor with 26, which means it is invertible. Taking the first two equations:

$$\begin{aligned} c_0 &= c_1^{-1} \pmod{26} \\ c_2 &= c_1^{-1} \pmod{26} \\ \Rightarrow c_0 &= c_2 \end{aligned}$$

Using the same reasoning we get $c_0 = c_2 = c_4 = c_6 = c_8$ and $c_1 = c_3 = c_5 = c_7 = c_9$. Now for every option of $c_0, \dots, c_8$ that has no common factor with 26 there is a possible character for $c_1, \dots, c_9$ that satisfies the above equations, given by $c_1 = c_0^{-1} \pmod{26}$. Therefore, there are 12 possible solutions to the equations, because we can choose c_0 to be any odd character different from 13 and all the other characters will be determined. Specifically, the pairs (c_0, c_1) that satisfy the equations are:

$$(1,1), (3,9), (5,21), (7,15), (9,3), (11,19), (15,7), (17,23), (19,11), (21,5), (23,17), (25,25).$$

7 Question 7

7.1 Question

Giovanni loves his car. He is driving it at a consistent, positive speed (but not faster than the speed limit of 90km/h). At one moment the car's computer shows 25952 km of mileage. Exactly 2 hours later, the mileage shown is again a palindrome number (a number that reads the same in both directions). How fast is Giovanni driving?

7.2 Answers

(A) `25 km/h`

(B) `50 km/h`

(C) `55 km/h`

(D) `63 km/h`

7.3 Proposed solution

The correct answer is `(C) 55km/h`.

Since Giovanni's speed is less than or equal to 90 km/h, he could have travelled at most 180 km in 2 hours, which corresponds to a maximum mileage of 26132 km. Between 25952 and 26132 there is only one palindrome number different from 25952, which is 26062. Therefore, since the speed is positive, the mileage shown by the computer after 2 hours must be 26062 km, meaning that the speed Giovanni was driving at was

$$s = \frac{26062 - 25952}{2}\ \texttt{km/h} = 55\ \texttt{km/h}.$$

8 Question 8

8.1 Question

A boy and a girl are talking. *"I am a boy"*, says the child with black hair. *"I am a girl"* says the one with blonde hair. At least one of them lied. Who is the boy and who is the girl?

8.2 Answers

(A) `The child with black hair is a girl, the one with blonde hair is a boy`

(B) `The child with black hair is a boy, the one with blonde hair is a girl`

(C) `Both the options are possible`

(D) `None of the options is possible`

8.3 Proposed solution

The correct answer is `(A) The child with black hair is a girl, the one with blonde hair is a boy`.

We know at least one of them is lying and they are one boy and one girl:

- If the black-haired child was the only one lying they would both be girls, which is not possible.
- If the blonde-haired child was the only one lying they would both be boys, which is not possible.
- If they were both lying, then the black-haired child would be a girl and the blonde-haired child would be a boy, which is a possible configuration therefore the correct answer.

9 Question 9

9.1 Question

The scores of the top 5 teams at CyberChallenge.IT have an interesting property. They form an arithmetic progression (the difference between a team's score and the previous one's score is constant) and the first 3 teams together have made 7 times the score of the other 2 teams together. Knowing that the sum of their scores is 3600, what is the score of the top team?

9.2 Answers

(A) `1300`

(B) `1340`

(C) `1380`

(D) `1420`

9.3 Proposed solution

The correct answer is `(C) 1380`.

Let's denote the scores of the teams with $a_1, a_2, \ldots, a_5$, where a_i is the score of the team at the i-th position. Since the scores form an arithmetic progression, we have $a_{i+1} = a_i - k$ for a fixed k. Expanding the relations, we have $a_i = a_1 - (i-1)k$. This means the sum of the first three scores is $a_1 + a_2 + a_3 = 3a_1 - 3k$ and the sum of the other two scores is $a_4 + a_5 = 2a_1 - 7k$. The total sum is $a_1 + \ldots + a_5 = 5a_1 - 10k$.
We can then write:

$$\begin{cases} a_1 + a_2 + a_3 = 7(a_4 + a_5) \\ a_1 + \ldots + a_5 = 3600 \end{cases} \quad \begin{cases} 3a_1 - 3k = 7(2a_1 - 7k) \\ 5a_1 - 10k = 3600 \end{cases} \quad \begin{cases} 11a_1 = 46k \\ a_1 - 2k = 720 \end{cases} \quad \begin{cases} 11(720 + 2k) = 46k \\ a_1 = 720 + 2k \end{cases}$$

$$\begin{cases} 7920 + 22k = 46k \\ a_1 = 720 + 2k \end{cases} \quad \begin{cases} 7920 = 24k \\ a_1 = 720 + 2k \end{cases} \quad \begin{cases} k = 330 \\ a_1 = 720 + 2 \cdot 330 = 1380 \end{cases}$$

Therefore, the solution is 1380.

10 Question 10

10.1 Question

Consider the following function.

```
function f(n):
  res = []
  while n > 0:
    t = n & 1
    n = n >> 1
    t = t + ((n & 1) << 1)
    n = n >> 1
    res.append(t) # adding t at the end of res
  return res
```

What is the result of `f(81721231)`?

10.2 Answers

(A) `[3, 3, 0, 2, 3, 1, 3, 3, 2, 3, 1, 3, 0, 1]`

(B) `[1, 0, 3, 1, 3, 2, 3, 3, 1, 3, 2, 0, 3, 3]`

(C) `[2, 2, 0, 1, 2, 1, 2, 2, 1, 2, 1, 2, 0, 1]`

(D) `[1, 0, 2, 1, 2, 1, 2, 2, 1, 2, 1, 0, 2, 2]`

10.3 Proposed solution

The correct answer is `(A) [3, 3, 0, 2, 3, 1, 3, 3, 2, 3, 1, 3, 0, 1]`.

At the end of each iteration of the `while` loop, `t` stores the lowest two bits of `n`, in the same order, and `n` itself is divided two times by 2. This means `res` contains the digits of `n` if written in base 4, in reverse order (the first element is the least significant digit, and so on). Since $81721231 \equiv 3 \pmod 4$, its least significant digit is 3, which narrows down the choice to just one answer, as only one array starts with a 3.

11 Question 11

11.1 Question

Consider the following function.

```
function f(n):
  res = 0
  for i = 0,...,n:
    t = n
    while t > 0:
      if i == t:
        res = res << 1
        res = res + ((i >> 1) & 1)
      t = t >> 1
  return res
```

What is the result of `f(20252025202520252025)`?

11.2 Answers

(A) `10126012601260126012`

(B) `10126012601260126013`

(C) `5063006300630063006`

(D) `5063006300630063007`

11.3 Proposed solution

The correct answer is `(A) 10126012601260126012`.

Inside the `while t > 0` loop, there is only one `t` value which respects `i == t`. In fact, said `l` the bit length of `n`, the `if` section will only be executed for `i` equal to `n >> x` for `x` in $\{$`l-1, l-2, ..., 1, 0`$\}$, in this order. So `res` will be set to the value `(n >> (x+1)) & 1`. Therefore, `res` and `n` will have the exact same bits except the last one, making `res =` $\lfloor n/2 \rfloor$. The answer is thus $\lfloor 20252025202520252025/2 \rfloor = 10126012601260126012$.

12 Question 12

12.1 Question

A company has 2025 computers. They are ordered in a row and numbered from 1 to 2025. One of them has been infected by a virus that every night moves to an adjacent computer **leaving** the previous one (so if it is in computer 3, it could move to 2 or 4, but if it is in computer 1, it can only move to computer 2; **note that only one computer at a time can be infected**). Every day, the only IT person of the company checks one and only one computer for viruses. If he follows an appropriate strategy, after how many days at most will he be sure to spot the virus?

12.2 Answers

(A) `2025`

(B) `4046`

(C) `4050`

(D) `He can never be sure`

12.3 Proposed solution

The correct answer is `(B) 4046`.

First of all, let's find a strategy that makes him find the virus. The strategy consists in checking computers number $2, 3, \ldots, 2024$ in this order for 2 times.

The first time we inspect computers from 2 to 2024, we start at day 1 from computer 2, which means we check even computers in odd days and odd computers in even days.

The second time we inspect computers from 2 to 2024, we start at day 2024 from computer 2, which means we check even computers during even days, and odd computers during odd days.

Let's suppose the virus starts at day 1 from an even computer. Then, in the odd days, it will always be infecting an even computer, while in the even days it will be infecting an odd computer. If this is the case, we will find the virus during our first inspection.

If at day 1 the virus starts from an odd computer instead, then it will be infecting even computers during even days and odd computers during odd days. If this is the case, we will find the virus during our second inspection.

Both the inspections will take at most $2023 \cdot 2 = 4046$ days.

Now, let's prove that we cannot do better. Let's suppose that there is a strategy that takes n days, with $n < 4046$. Then there will be at least one computer in position k with $2 \leq k \leq 2024$ that will only be checked once, which means only during an odd day or only during an even day. Let's suppose that this computer is checked only during an odd day (the case with an even day is analogous). During even days the virus can hide in computer k, while during the odd ones it will be either in $k-1$ or in $k+1$. The IT person cannot check both $k-1$ and $k+1$ in a single day, so the virus can hide in the not-checked one. The IT person cannot therefore be sure to find it.

Summing up, each computer from 2 to 2024 must be checked during both an odd and an even day, thus the strategy must take at least $2023 \cdot 2 = 4046$ days.

13 Question 13

13.1 Question

Consider the following function.

```
1 function f(n):
2   res = 0
3   for i = 0,...,n-1:
4     res = (res << 2) - res + 1
5   return res
```

How many positive integers `x` are there such that `f(x) < 9`^{100}?

13.2 Answers

(A) `100`

(B) `200`

(C) `1024`

(D) `59049`

13.3 Proposed solution

The correct answer is `(B) 200`.

At each iteration of the `for` loop, `res` is multiplied by 4 with the left shift, then subtracted from the result (ending up in a multiplication by 3) and 1 is added to it. In other words, we can rewrite the fourth line of code as `res = res*3 + 1`.

If we encode `res` in base 3, it will look like $11\dots11$, with `n` repetitions of the digit 1. For example, $f(3) = 111_3$[1] and $f(7) = 1111111_3$. Let's point out that $9 = 100_3$, so $9^{100} = 100\dots00_3$ with two hundreds 0 digits. Since `f(n)` is written with `n` digits, this means that $f(200) < 9^{100} < f(201)$.

We conclude that the answer is 200.

[1] 111_3 indicates the number 111 in base 3, which is equal to 13 in base 10.

14 Question 14

14.1 Question

Consider the following function.

```
function f(n):
  res = 0
  for i = 0,...,n-1:
    res = (res + 1) / 2
  return res
```

Supposing that the function is executed in an environment that supports infinite decimal precision, what is the lowest value for `n` such that `1-f(n)` is less than or equal to 8^{-200}?

14.2 Answers

(A) `200`

(B) `400`

(C) `600`

(D) `800`

14.3 Proposed solution

The correct answer is `(C) 600`.

At each iteration of the `for` loop, the new value of `res` is the average between its old value and 1. So, the distance between `res` and 1, expressed by `1-f(n)`, is halved. We can rewrite this information as $1 - f(n) = \left(\frac{1}{2}\right)^n$.

Thus, we have $8^{-200} = \left(\frac{1}{8}\right)^{200} = \left(\frac{1}{2}\right)^{600} = 1 - f(600)$.

The answer is 600.

15 Question 15

15.1 Question

4 people are at a hacking conference. Each of them is either a white-hat, a black-hat or a grey-hat. White-hats wear a white hat and always tell the truth, black-hats wear a black hat and always lie, while grey-hats wear a white hat, but they always lie. You know that there is exactly one grey-hat, but you don't know anything about the others. You can only ask a person A the color of the hat **another** person B is wearing, and **you cannot see their hats**. How many questions will you have to ask to be sure to find the grey-hat?

15.2 Answers

(A) `3`

(B) `4`

(C) `5`

(D) `You are never sure to find him`

15.3 Proposed solution

The correct answer is `(B) 4`.

First of all, let's analyse what happens when we ask person A what person B is wearing and then we ask person B what person A is wearing.

- If they are both white-hats or both black-hats then both answers will be *White*.
- If they are one white-hat and one black-hat, both answers will be *Black*.
- If one between A and B is a grey-hat, one person will answer *White* and one will answer *Black*.

Let A, B, C, D be the four people. We first ask A about B and vice versa. Following the reasoning above, if they give different answers, we know that the grey-hat is one between A and B, if they give the same answer, the grey-hat is one between B and C.

We can repeat the same questions with A and C, so we discover if the grey-hat is one between A and C or one between B and D. Intersecting this information with the one obtained before, we are able to uniquely identify the grey-hat using only 4 questions.

Now, let's prove it is not possible with only 3 questions. If two of the questions are reciprocal, let's say between A and B, we know that the grey-hat is either between A and B or between C and D, depending on the answers, but the third question is not enough to identify the grey-hat uniquely. Instead, if we ask no reciprocal questions, the possibilities for the grey-hat will still cover all 4 people. By supposing that one of the people is the grey-hat, we could select the type of hats for the other people in a consistent way with the answers. Thus there is no way to identify the grey-hat with only 3 questions.